Computer Graphics

Chapter_4

2D Transformations

Lecture #3

Numerical Examples

- Example (1):

Translate the triangle with the vertices $V_1(1,1)$, $V_2(3,1)$ & $V_3(2,3)$ using the translation parameters $t_x = 4$, $t_y = 3$.

Solution:

Use the translation matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \boldsymbol{t_x} \\ 0 & 1 & \boldsymbol{t_y} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Use $t_x = 4$, $t_y = 3$, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \boldsymbol{4} \\ 0 & 1 & \boldsymbol{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Step 1: Translated V_1

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \mathbf{4} \\ 0 & 1 & \mathbf{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \mathbf{4} \\ 0 & 1 & \mathbf{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 0 \times 1 + 4 \times 1 \\ 0 \times 1 + 1 \times 1 + 3 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$

Step 2: Translated V_2

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \mathbf{4} \\ 0 & 1 & \mathbf{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

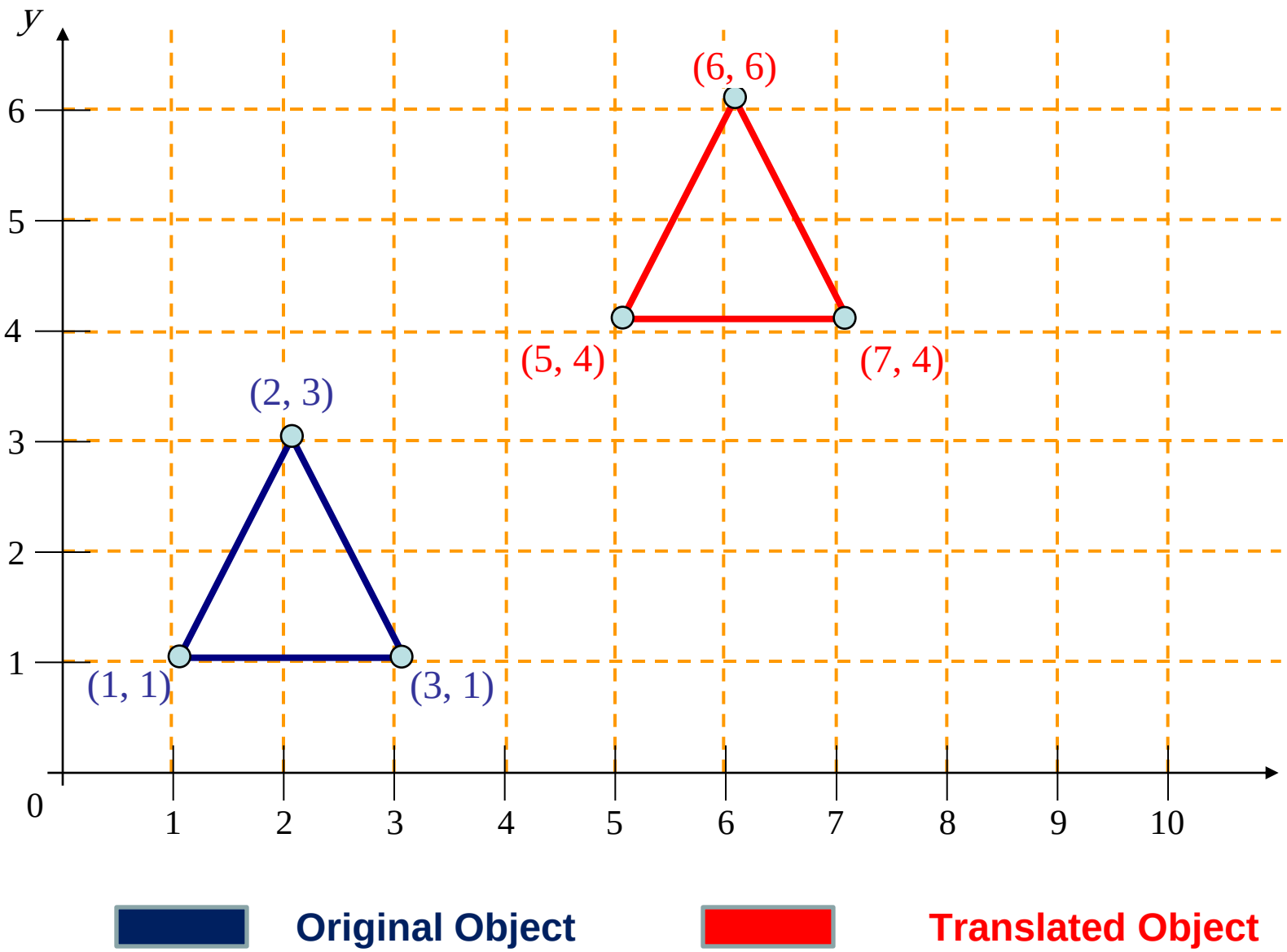
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \mathbf{4} \\ 0 & 1 & \mathbf{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \times 3 + 0 \times 1 + 4 \times 1 \\ 0 \times 3 + 1 \times 1 + 3 \times 1 \\ 0 \times 3 + 0 \times 1 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}$$

Step 3: Translated V_3

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \mathbf{4} \\ 0 & 1 & \mathbf{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \mathbf{4} \\ 0 & 1 & \mathbf{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \times 2 + 0 \times 3 + 4 \times 1 \\ 0 \times 2 + 1 \times 3 + 3 \times 1 \\ 0 \times 2 + 0 \times 3 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 1 \end{bmatrix}$$



Original Object **Translated Object**

- Example (2):

Magnify the triangle with the vertices $V_1(1,1)$, $V_2(3,1)$ & $V_3(2,3)$ using the scaling parameters $S_x = 2$, $S_y = 1.5$.

Solution:

The scaling matrix is:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & \mathbf{0} \\ 0 & S_y & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Use $S_x = 2$, $S_y = 1.5$, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & \mathbf{0} \\ 0 & 1.5 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Step 1: Scaled V_1:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & \mathbf{0} \\ 0 & 1.5 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & \mathbf{0} \\ 0 & 1.5 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 0 \times 1 + 0 \times 1 \\ 0 \times 1 + 1.5 \times 1 + 0 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1.5 \\ 1 \end{bmatrix}$$

Step 2: Scaled V_2:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & \mathbf{0} \\ 0 & 1.5 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

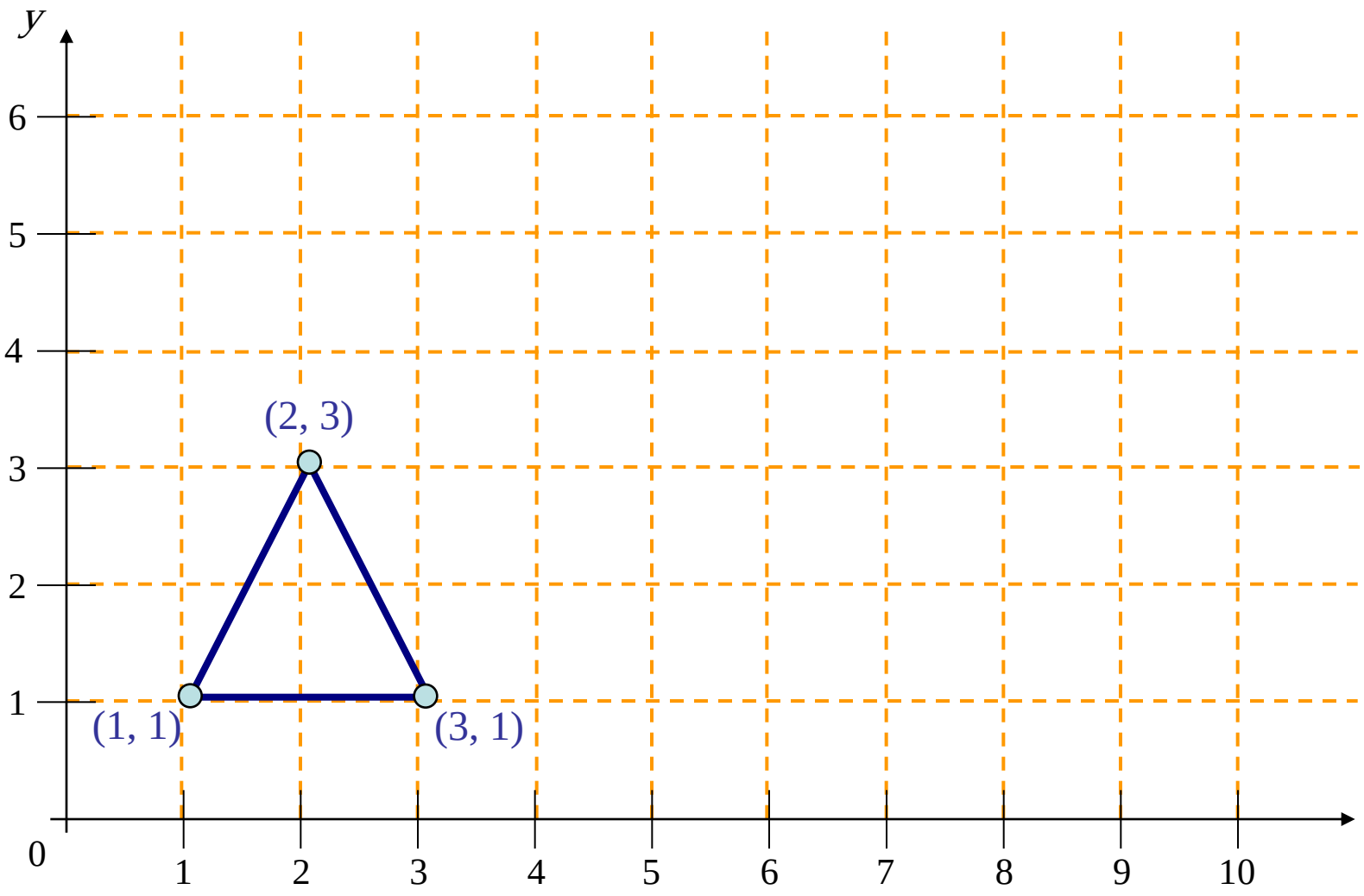
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & \mathbf{0} \\ 0 & 1.5 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \times 3 + 0 \times 1 + 0 \times 1 \\ 0 \times 3 + 1.5 \times 1 + 0 \times 1 \\ 0 \times 3 + 0 \times 1 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 1.5 \\ 1 \end{bmatrix}$$

Step 2: Scaled V_3:

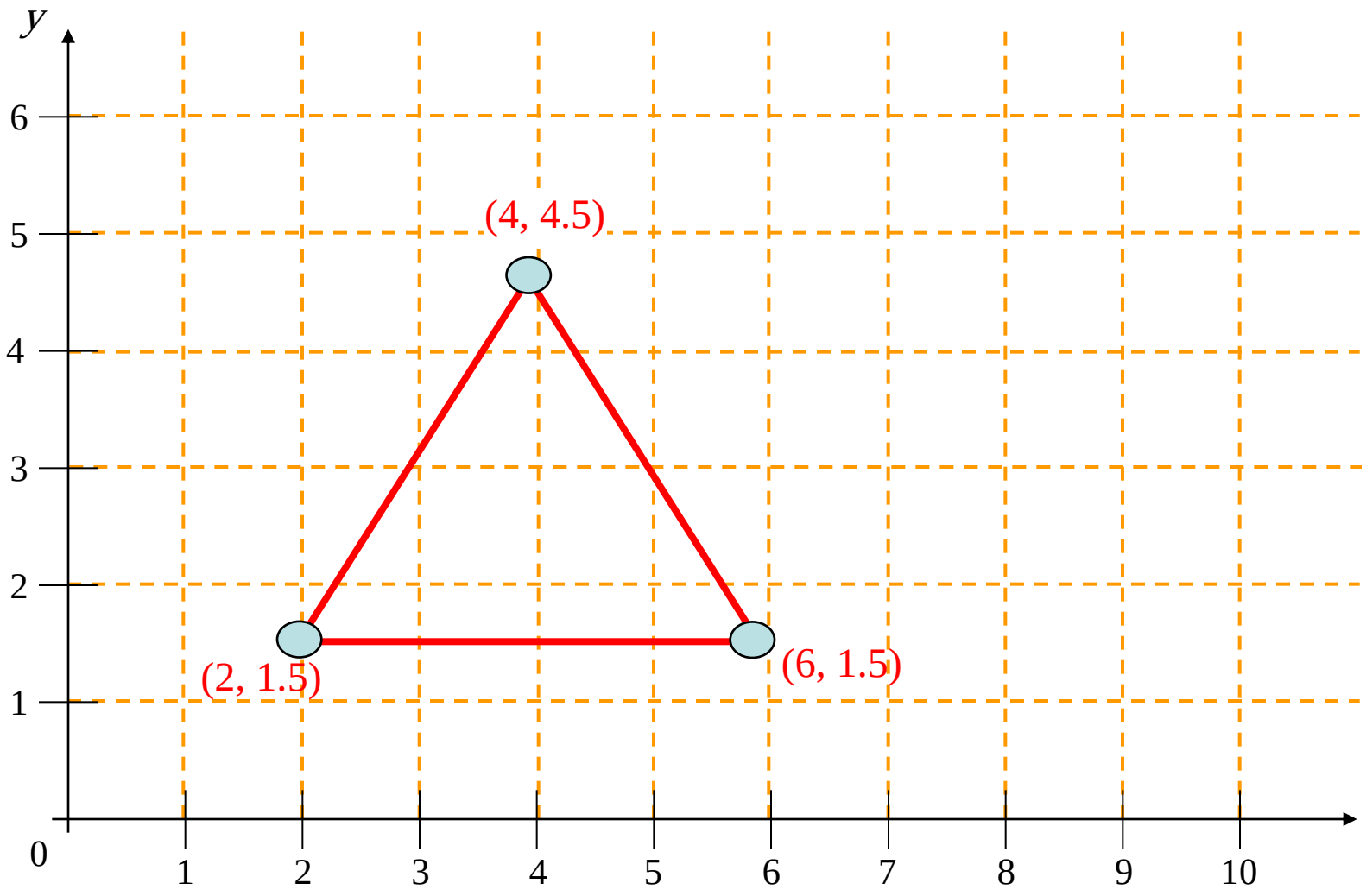
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & \mathbf{0} \\ 0 & 1.5 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & \mathbf{0} \\ 0 & 1.5 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \times 2 + 0 \times 3 + 0 \times 1 \\ 0 \times 2 + 1.5 \times 3 + 0 \times 1 \\ 0 \times 2 + 0 \times 3 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4.5 \\ 1 \end{bmatrix}$$



Original Object



Scaled Object

- Example (3):

Rotate the object with the vertices $V_1(0,0)$, $V_2(2,0)$, $V_3(2,2)$, $V_4(1,3)$, & $V_5(0,2)$ using a rotation angle 45^0 in the clockwise direction .

Solution:

The rotation matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & \mathbf{0} \\ \sin\theta & \cos\theta & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Use the rotation angle, -45^0, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Step 1: Rotate V_1:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos 45^0 \times 0 + \sin 45^0 \times 0 + 0 \times 1 \\ -\sin 45^0 \times 0 + \cos 45^0 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Step 2: Rotate V_2:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos 45^0 \times 2 + \sin 45^0 \times 0 + 0 \times 1 \\ -\sin 45^0 \times 2 + \cos 45^0 \times 0 + 0 \times 1 \\ 0 \times 2 + 0 \times 0 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{2}} \\ \frac{-2}{\sqrt{2}} \\ 1 \end{bmatrix}$$

Step 3: Rotate V_3:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos 45^0 \times 2 + \sin 45^0 \times 2 + 0 \times 1 \\ -\sin 45^0 \times 2 + \cos 45^0 \times 2 + 0 \times 1 \\ 0 \times 2 + 0 \times 2 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{\sqrt{2}} \\ 0 \\ 1 \end{bmatrix}$$

Step 4: Rotate V_4:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos 45^0 \times 1 + \sin 45^0 \times 3 + 0 \times 1 \\ -\sin 45^0 \times 1 + \cos 45^0 \times 3 + 0 \times 1 \\ 0 \times 1 + 0 \times 3 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} \\ 1 \end{bmatrix}$$

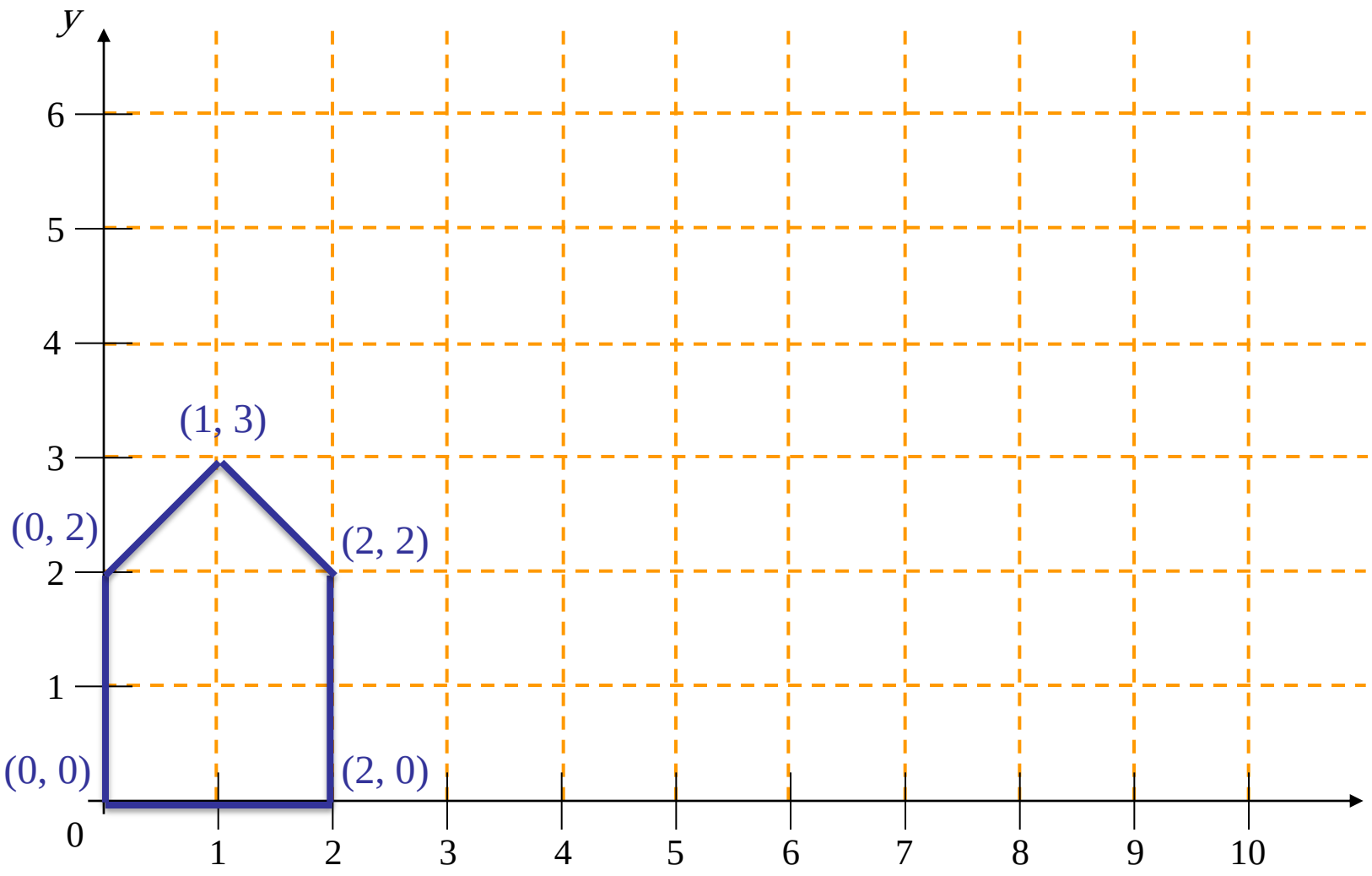
Step 5: Rotate V_5:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

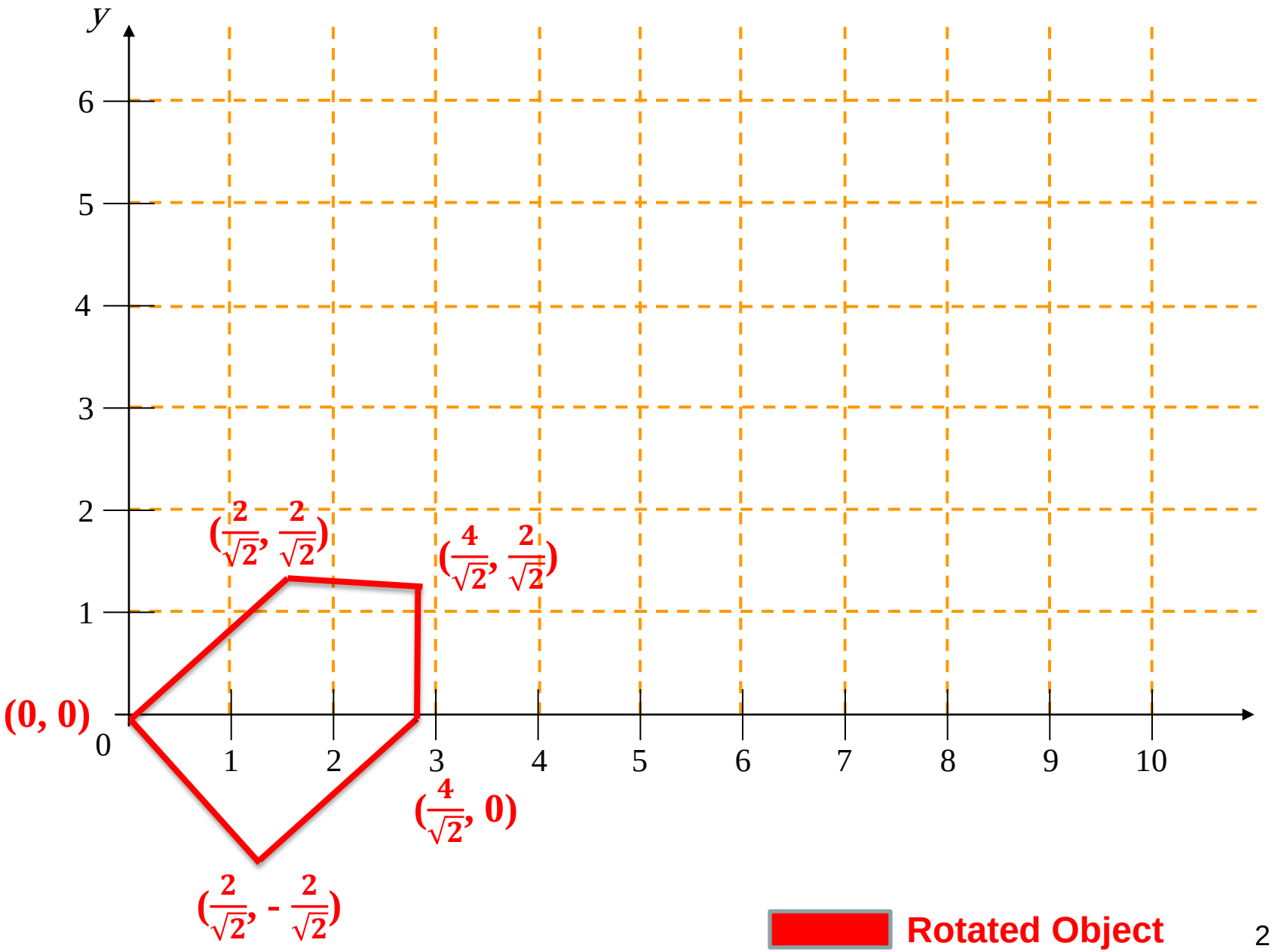
Recall matrix multiplication, then:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & \sin 45^0 & \mathbf{0} \\ -\sin 45^0 & \cos 45^0 & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} =$$

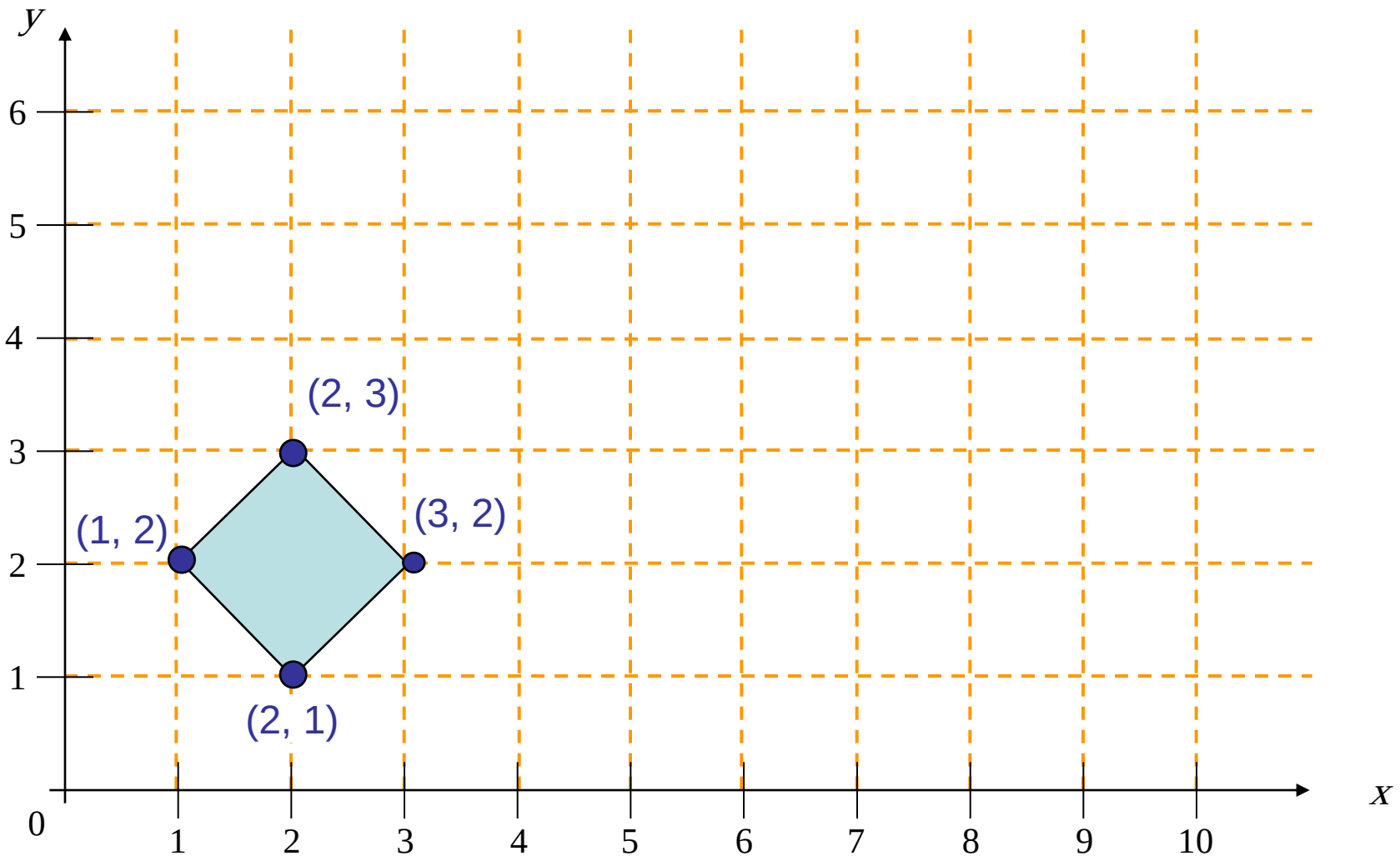
$$\begin{bmatrix} \cos 45^0 \times 0 + \sin 45^0 \times 2 + 0 \times 1 \\ -\sin 45^0 \times 0 + \cos 45^0 \times 2 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} \\ 1 \end{bmatrix}$$



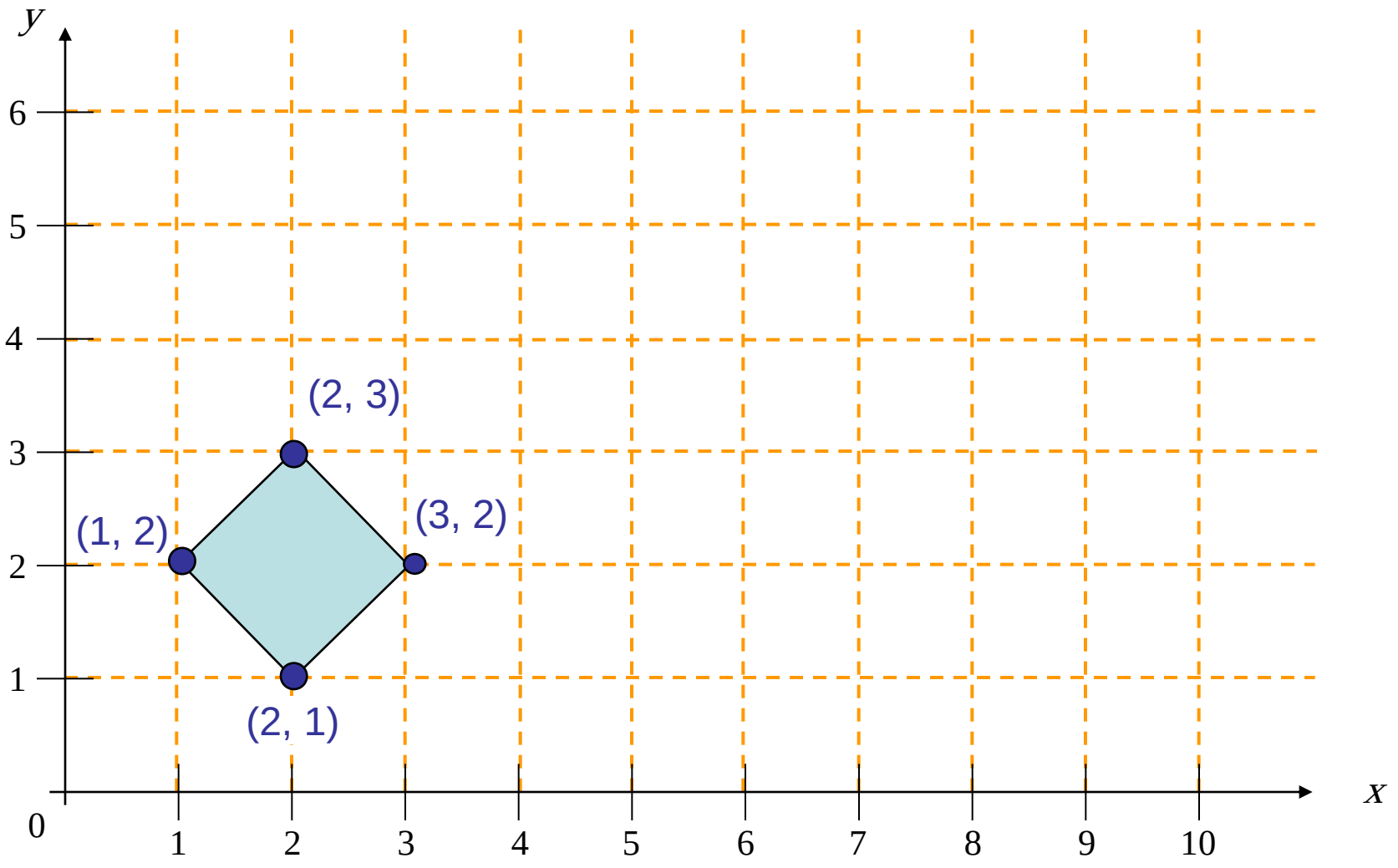
Original Object


y
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
(0, 0)
$(\frac{2}{\sqrt{2}}, \frac{2}{\sqrt{2}})$
$(\frac{4}{\sqrt{2}}, \frac{2}{\sqrt{2}})$
$(\frac{4}{\sqrt{2}}, 0)$
$(\frac{2}{\sqrt{2}}, -\frac{2}{\sqrt{2}})$
Rotated Object

Exercises 1: Translate the shape below by (7, 2)



Exercises 2: Scale the shape below by 3 & 2 in the *x*- and *y*-directions



Exercises 3: Rotate the shape below by 30° about the origin.